

I Linear Analysis I

Abstract Vector Spaces

1.1	Vector Algebra
1.2	Spans and Linear Independence
1.3	Products, Sums, and Complements
1.4	Dimension, Replacement, and Extension
1.5	Quotient Spaces
	Exercises

Linear Transformations and Matrices

2.1	Basics of Linear Transformations I
2.2	Basics of Linear Transformations II
2.3	Rank, Nullity, and the First Isomorphism Theorem
2.4	Matrix Representations
2.5	Composition, Change of Basis, and Similarity
2.6	Important Example: Bernstein Polynomials
2.7	Linear Systems
2.8	Determinants I
2.9	Determinants II
	Exercises

Inner Product Spaces

3.1	Introduction to Inner Products
3.2	Orthonormal Sets and Orthogonal Projections
3.3	Gram–Schmidt Orthonormalization
3.4	QR with Householder Transformations
3.5	Normed Linear Spaces
3.6	Important Norm Inequalities
3.7	Adjoins
3.8	Fundamental Subspaces
3.9	Least Squares

Exercises

Spectral Theory

4.1 Eigenvalues and Eigenvectors
4.2 Invariant Subspaces
4.3 Diagonalization
4.4 Schur's Lemma
4.5 The Singular Value Decomposition
4.6 Consequences of the SVD
Exercises

II Nonlinear Analysis I

Metric Space Topology

5.1 Metric Spaces and Continuous Functions
5.2 Continuous Functions and Limits
5.3 Closed Sets, Sequences, and Convergence
5.4 Completeness and Uniform Continuity
5.5 Compactness
5.6 Uniform Convergence and Banach Spaces
5.7 The Continuous Linear Extension Theorem
5.8 Topologically Equivalent Metrics
5.9 Topological Properties
5.10 Banach-valued Integration
Exercises

Differentiation

6.1 The Directional Derivative
6.2 The Fréchet Derivative in \mathbb{R}^n
6.3 The General Fréchet Derivative
6.4 Properties of Derivatives
6.5 Mean Value Theorem and Fundamental Theorem of Calculus
6.6 Taylor's Theorem
Exercises

Contraction Mappings and Applications

7.1 Contraction Mapping Principle
7.2 Uniform Contraction Mapping Principle
7.3 Newton's Method
7.4 The Implicit and Inverse Function Theorems
7.5 Conditioning
Exercises

III Nonlinear Analysis II

Integration I

8.1 Multivariable Integration

8.2	Overview of Daniell-Lebesgue Integration
8.3	Measure Zero and Measurability
8.4	Monotone Convergence and Integration on Unbounded Domains
8.5	Fatou's Lemma and the Dominated Convergence Theorem
8.6	Fubini's Theorem and Leibniz' Integral Rule
8.7	Change of Variables
	Exercises

***Integration II**

9.1	Every Normed Space Has a Unique Completion
9.2	More about Measure Zero
9.3	Lebesgue-Integrable Functions
9.4	Proof of Fubini's Theorem
9.5	Proof of the Change of Variables Theorem
	Exercises

Calculus on Manifolds

10.1	Curves and Arclength
10.2	Line Integrals
10.3	Parametrized Manifolds
10.4	*Integration on Manifolds
10.5	Green's Theorem
	Exercises

Complex Analysis

11.1	Holomorphic Functions
11.2	Properties and Examples
11.3	Contour Integrals
11.4	Cauchy's Integral Formula
11.5	Consequences of Cauchy's Integral Formula
11.6	Power Series and Laurent Series
11.7	The Residue Theorem
11.8	*The Argument Principle and its Consequences
	Exercises

IV Linear Analysis II

Spectral Calculus

12.1	Projections
12.2	Generalized Eigenvectors
12.3	The Resolvent
12.4	Spectral Resolution
12.5	Spectral Decomposition I
12.6	Spectral Decomposition II
12.7	Spectral Mapping Theorem
12.8	The Perron-Frobenius Theorem
12.9	The Drazin Inverse
12.10	*Jordan Canonical Form

Exercises

Iterative Methods

- 13.1 Methods for Linear Systems
- 13.2 Minimal Polynomials and Krylov Subspaces
- 13.3 The Arnoldi Iteration and GMRES Methods
- 13.4 *Computing Eigenvalues I
- 13.5 *Computing Eigenvalues II
- Exercises

Spectra and Pseudospectra

- 14.1 The Pseudospectrum
- 14.2 Asymptotic and Transient Behavior
- 14.3 *Proof of the Kreiss Matrix Theorem
- Exercises

Rings and Polynomials

- 15.1 Definition and Examples
- 15.2 Euclidean Domains
- 15.3 The Fundamental Theorem of Arithmetic
- 15.4 Homomorphisms
- 15.5 Quotients and the First Isomorphism Theorem
- 15.6 The Chinese Remainder Theorem
- 15.7 Polynomial Interpolation and Spectral Decomposition
- Exercises