

I Dynamic Modeling I

Modeling

- 1.1 The Art and Science of Mathematical Modeling
- 1.2 Examples of Dynamic Models: Harmonic motion
- 1.3 Examples of Dynamic Models: population dynamics
- Exercises

Existence and Uniqueness

- 2.1 Review of Ordinary Differential Equations
- 2.2 Existence and Uniqueness
- 2.3 Continuation and Well Posedness
- 2.4 Homogeneous Linear Systems
- 2.5 Matrix Exponentials and Fundamental Matrices
- 2.6 Non-Homogeneous Linear Systems
- Exercises

Stability Theory

- 3.1 Stability of Linear Systems
- 3.2 Stability of Autonomous Systems
- 3.3 Examples
- 3.4 Structural stability
- 3.5 Lyapunov's method
- 3.6 *More Lyapunov Stability
- 3.7 Poincare-Bendixson Theorem
- 3.8 *Stable Manifold Theorem
- 3.9 *Center Manifold Theory
- Exercises

Bifurcation Theory

- 4.1 What is a bifurcation?
- 4.2 Bifurcations of co-dimension 1
- Exercises

II Partial Differential Equations

Introduction

5.1	Introduction to PDEs	
5.2	General form of a PDE	
5.3	Linearity	
5.4	Classification of 2nd order PDEs in dimension 1	
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Hyperbolic PDE

6.1	Conservation Laws	
6.2	Method of Characteristics	
6.3	Auxiliary conditions and the method of characteristics	
6.4	A canonical hyperbolic PDE: the wave equation	
6.5	Equation of motion for a vibrating string	
6.6	Traveling waves	
6.7	Standing/ plane waves	
	Exercises	

Auxiliary Conditions, Well-Posedness, and Parabolic and Elliptic equations

7.1	Auxiliary conditions	
7.2	Well-posedness	
7.3	Parabolic equations: the canonical example of the heat equation	
7.4	Equilibrium/Elliptic Equations	
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Eigenfunction Expansions

8.1	Exponentials of linear differential operators	
8.2	Sturm-Liouville Problems	
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Green's Function

9.1	Construction of the Green's function via eigenfunction expansion	
9.2	Distributions	
9.3	Distributions and PDE's	
9.4	Computation of the Green's Function	
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III Calculus of Variations

Introduction to Optimization and the Calculus of Variations: background material

10.1	Introductory concepts	
10.2	Formalizing the Optimization Process	
10.3	Gateaux differential	
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The Simplest Problem

11.1	Formal Derivation of Euler's Equation	
11.2	Rigorous justification without integration by parts	

11.3	Examples	
11.4	Special Cases	
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Generalizations of the Simplest Problem

12.1	Variable endpoints	
12.2	Changes to natural boundary conditions	
12.3	Functionals of several variables	
12.4	Higher dimensional objects	
12.5	Further Generalizations	
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Constraints

13.1	Introduction to constraints	
13.2	Integral (Isoperimetric) Constraints	
13.3	Non-Integral (Finite) Constraints	
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Hamilton's Principle

14.1	Introduction to Hamilton's principle	
14.2	Some physical examples	
14.3	Hamiltonian description	
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Symmetry and Conservation: Noether's Theorem

15.1	Introduction and background material	
15.2	Noether's Theorem for one independent variable	
15.3	Generalization of Noether's Theorem	
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Necessary AND Sufficient Conditions for Weak Maxima/Minima

16.1	Necessary Conditions to distinguish between maxima/minima	
16.2	Sufficient Conditions for a maximum/minimum	
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Strong Extrema

17.1	Strong versus Weak	
17.2	Weirstrass-Erdmann corner conditions	
17.3	Weirstrass function and necessary conditions for a minimum/ maximum	
17.4	Sufficient conditions for strong extrema	
17.5	Final Discussion	
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IV Optimal Control

Introduction to Optimal Control

	Exercises	
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Formal Derivation of Pontryagin’s Maximum Principle
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19.2 Pontryagin’s maximum principle
19.3 Examples
Exercises

Bang-bang and singular control problems
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Different forms of the cost-functional
21.1 Bolza cost functional with a fixed end time
21.2 Bolza cost functional with the final time varying
21.3 Mayer form of the cost functional
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Linear Quadratic Regulator (the ‘right’ way to optimize)
22.1 Introduction
22.2 Derivation of LQR
22.3 Summary and implementation of LQR
22.4 Infinite Horizon and the inverted pendulum
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Inequality constraints
23.1 Introduction and motivation
23.2 Formal derivation of the KKT conditions
23.3 Some examples
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Hamilton Jacobi Bellman Equation
24.1 The Value Function
24.2 Derivation of the HJB equation
24.3 The HJB equation and sufficiency conditions
Exercises

V Optimal Control Theory

Mathematical Systems
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25.2 Mathematical Systems
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Control Theory: Discrete Case
26.1 Reachability
26.2 Observability

Linear Control Theory
27.1 Continuous Linear-Time Invariant Systems
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27.3	Transfer functions
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27.5	Canonical Decomposition
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Optimal Control

28.1	The Problem and Notation
28.2	The Maximum Principle
28.3	The Adjoint Equation
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28.9	Free Terminal Time
28.10	Current Value Formulation
28.11	Summary of Current Value Conditions
28.12	Interpretation of the Adjoint
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